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Identification of Cutting Coefficients from Multiple Milling Tests

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Abstract

The simulation of cutting forces in milling is a prerequisite for reliable process modeling. Among the different approaches used in the literature, the mechanistic models show good compromise between good precision and reasonable simulation time. The main challenge for these models is the identification of their parameters, such as the cutting coefficients, for a given tool/material couple.

This paper presents a method based on an inverse analysis to identify these parameters. At first, the preprocessing of the signal is made to automatically detect the time during which the cutter is actively engaged in the workpiece. The consistency of the input data is also checked to reject outliers. Then, to avoid the classical pitfalls of this approach such as the non-uniqueness of solution, the identification is made on a whole database of results using an iterative method. The use of optimization algorithm allows the identification of parameters having nonlinear effect on the results such as cutter runout.

A set of 57 milling tests in Ti6Al4V alloys have been used to demonstrate the effectiveness of this method. It allows a reduction of 5 to 20% of the root mean square error between the model and the measurements as compared to the use of coefficients identified on a single cutting test.

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Keywords: Cutting forces ; Milling; Mechanistic model; Identification

1. Introduction

Accurate computation of cutting forces during machining operation is a key aspect for a reliable simulation of the whole process. Chatter vibration is an example of problem that can be modelled and control based on predictive models [1, 2, 3]. Mechanistic models are often selected for their ease of use and reliability [4, 5, 6]. The drawback of this method is that the input parameters of the cutting force model are often difficult to find out from intrinsic properties of the materials such as Young's modulus, yield strength or hardness.

This disadvantage can be resolved using inverse analysis method that allows the identification of the cutting forces model parameters from the recording of the cutting forces during a machining operation.

An inverse analysis procedure has been previously developed, able to identify, in addition to the cutting force model parameters, unknown parameters that affect the cutting process such as runout [7] for a single operation. This approach based on the instantaneous signal of cutting force can include more precise model of the cutter geometry or cutting force model than methods based on averaged forces [8] as highlighted in 2212-8271 © 2025 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

[9]. For example, it is possible to take into account a nonlinear evolution of the specific pressure with respect to the actual undeformed chip thickness.

The present paper proposes a method to extend the concept to the identification of cutting force model parameters using a set of experimental test to enhance the reliability of the identification. It allows, among others, the problem of the nonuniqueness of the solution if a single cutting condition is selected for the identification. The paper will describe the inverse analysis procedure followed by the two-pass optimisation loop. The method will then be used on a database of milling tests performed on a titanium alloy to compare the quality of the fit as compared to previous method of identification.

2. Description of the method

The inverse analysis procedure is divided in several steps. At first, a preprocessing of the signal intended to correct the possible drift of the piezoelectric sensor, extract the useful part of the signal (while the cutter is actually machining the part) and find the actual spindle speed to average the cutting forces along all revolutions of the cutter.

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Then inverse analysis method is applied obtains a first guess of the parameters having nonlinear effect (initial shift of the cutter, runout of the cutter,...) on each signal individually. A multipass optimisation algorithm is applied to the whole database which at first gets a first estimation of the cutting force model parameters considering these parameters remians constant, then frees then to refine the results of the previous steps. The global procedure is summarized in figure 1.



Fig. 1. Algorithm for the identification

2.1. Preprocessing

The typical cutting force measurement made using a piezoelectric dynamometer measures different phases (figure 2) of the cutting process: 1) an initial phase when the cutter is not in contact with the workpiece 2) a transition phase when the immersion angle gradually increase to its nominal value 3) the useful part of the signal during which the cutting parameters remain constant 4) an exit phase when the cutter progressively loses its contact with the workpiece. These can be repeated several times if the recording includes multi-pass machining operation. A preprocessing phase is thus necessary to extract the useful part of the signal automatically.

2.1.1. Offset and useful signal extraction

For the offset of the sensor, the mathematical mean value of cutting force along x, y and z axis is computed during phase 1 of the recording and is subtracted from the measured force along the respective axis. The recording time is then divided in m sets of n time intervals (selected such as each interval accounts for hundred revolutions of the tool) and the root mean square (RMS) value of the signal is evaluated as:

$$RMS_{j} = \sqrt{\sum_{i=1}^{n} \left(F_{j,i} - \mu_{j}\right)^{2}}$$
(1)



Fig. 2. Cutting force measurement pattern

where $F_{j,i}$ is the *i*th force value in the *j*th time interval and μ_j the arithmetic mean of *F* over the *j*th interval. A threshold is then use to determine the intervals for which RMS is significantly different from zero (thus the tool is machining the workpiece). A convenient value of the threshold is the value of 5 time the RMS value of the signal recorded at the first seconds of the measurement file (so while the cutter is out of the material.

Finally, the entry and exit phase of the signal are removed from the intervals. a second treshold is determined as:

$$treshold = \mu_{RMS} - \sigma_{RMS}/2 \tag{2}$$

with μ_{RMS} and σ_{RMS} the mean value and the standard deviation of the root mean square (RMS) value of cutting forces during the phase identified as into the material.

2.1.2. Actual spindle speed estimations and averaging

As the identification method is based on the temporal evolution of the cutting force, the exact knowledge of the spindle speed used for the machining operation is crucial. However, the real spindle speed may slightly differs from the programmed, causing potential accumulated errors during the identification procedure. To hinder this problem, an averaging technique is used to identify precisely the actual cutting speed by minimizing the RMS difference of the cutting force signal over successive cutter revolutions [10].

Figures 3 and 4 show the superimposition of the cutting force along x direction for each tool revolution while the spindle speed has been correctly identified (figure 3) and while the prediction is incorrect (figure 4).

2.2. Inverse analysis

this phase allows the identification of a first guess of the initial shift α (angular position of the first tooth of the cutter at initial time of the recording) and runout parameters ρ and λ of the cutter for each individual recording.



Fig. 3. Superposition of signals while the spindle speed is correct



Fig. 4. Superposition of signals while spindle speed is incorrect

The mill is divided in n_d disks along its axis. On each disk the infinitesimal forces along tangential (*t*), radial (*r*) and axial (*a*)are computed using the model proposed by Altintas [1]:

$$\begin{cases} dF_t = K_{te} \cdot dS + K_{tc} \cdot h \cdot db \\ dF_r = K_{re} \cdot dS + K_{rc} \cdot h \cdot db \\ dF_a = K_{ae} \cdot dS + K_{ac} \cdot h \cdot db \end{cases}$$
(3)

h is the undeformed chip thickness, *db* is the projected length of an infinitesimal cutting flute in the direction along the cutting velocity and *dS* the local cutting edge length). The coefficients $K_{.c}$ are the specific pressure (linked to the shearing of the chip) that needs to be identified. The edges coefficients $K_{.e}$ (linked to the edge forces) can be included in the model depending on the choice of the user or assume to be equal to zero. The identification method rewrites equation 3 into a matrix relationship between cutting forces and cutting coefficients:

$$\begin{cases}
\frac{dF_t}{dF_r} \\
\frac{dF_a}{dF_a}
\end{cases} = [A] \cdot \overbrace{\begin{cases} K_{tc} \\ K_{rc} \\ K_{ac} \\ K_{re} \\ K_{ae} \\ K_{ae} \\ K_{ae} \\ \end{bmatrix} or = [A] \cdot \overbrace{\begin{cases} K_{tc} \\ K_{tc} \\ K_{ac} \\ K_{ac} \\ \end{bmatrix}}^{(K)}$$
(4)

$$[A] = \begin{bmatrix} h \cdot db & 0 & 0 & dS & 0 & 0 \\ 0 & h \cdot db & 0 & 0 & dS & 0 \\ 0 & 0 & h \cdot db & 0 & 0 & dS \end{bmatrix}$$
(5)

The forces are projected in the reference frame of the measurement device. The classical transformation matrix [*B*] performs the projection (κ is the axial immersion angle).

$$[B] = \begin{bmatrix} -\cos\theta - \sin\theta \cdot \sin\kappa & -\sin\theta \cdot \cos\kappa \\ \sin\theta & -\cos\theta \cdot \sin\kappa & -\cos\theta \cdot \cos\kappa \\ 0 & -\cos\kappa & -\sin\kappa \end{bmatrix}$$
(7)

The immersion angle θ takes three parameters into account:

- rotation of the tool ($\Omega \cdot dt$ with Ω the spindle speed);
- shift of each cutting edge around the tool $(\frac{2\pi}{Z}$ for a tool with Z edges and uniform pitch);
- shift of the cutting edge due to helix angle (^{2ztani}/_D for a cylindrical mill, see [5] for other geometries);

These relationships are then added for each tooth and each slice to perform numerical integration along the cutting edges:

$$\begin{cases}
 F_x \\
 F_y \\
 F_z
\end{cases} = \sum_{i=1}^{n_d} \sum_{j=1}^{Z} \begin{cases}
 dF_x(i, j) \\
 dF_y(i, j) \\
 dF_z(i, j)
\end{cases}$$

$$= \overbrace{\left(\sum_{i=1}^{n_d} \sum_{j=1}^{n_t} [B] \cdot [A]\right)}^{[C]} \cdot \{K\}$$
(8)

Matrix [*C*] (dimension 3x6 or 3x3 depending on the selected cutting force model) links cutting coefficients to cutting forces. At each time step a matrix $[C^k]$ can be build (*k* is the index of the current time step). This method is similar to the work of Ko and Cho [6] but instead of finding the cutting coefficients for each time step, all the matrices are [*C*] assembled to get a global system:

$$\begin{array}{c}
\begin{bmatrix}
F_{1} \\
F_{x} \\
F_{y} \\
F_{z} \\
F_{z} \\
F_{z} \\
F_{z} \\
\hline
\vdots
\end{array}
\right) =
\begin{bmatrix}
D \\
C^{1} \\
C^{1} \\
C^{2} \\
\hline
C^{2} \\
\hline
\vdots
\end{array}
\right) \cdot \{K\}$$
(9)

The cutting forces parameters can be computed by filling the vector $\{F\}$ with the measured forces and by applying least square fitting method to solve the overdetermined system:

$$[K] = \left([D]^T [D] \right)^{-1} \cdot \left([D]^T [F] \right)$$
(10)

[K] is the matrix containing the six unknown coefficients, [D], the assembly of all $[C^k]$ matrices and [F] the measured cutting forces.

While the cutting coefficients are obtained, the cutting forces can be simulated and the quality of the fitting can be estimated by the root mean square error between computed (F_c) and measured effort (F_m) :

$$RMS_{error} = \frac{\sqrt{\sum_{i=0}^{npoints} \left(\left(F_c^i - F_m^i \right) \cdot \Delta \theta \right)^2}}{\theta_{end} - \theta_{begin}}$$
(11)

This indicator is used to determine the parameters that have a nonlinear effect on the computation of the cutting forces such as the initial shift of the cutter α and the runout of the cutter ρ and α [7].

2.3. Optimisation

Optimisation procedure works in three steps:

- all cutting force measurements are assembled in a single numerical system to get a single set of cutting force model parameters for the whole database;
- the values of runout and initial shift are refined using an optimisation algorithm to get the final coefficients of the cutting force model.

2.4. Global analysis

Based on the inverse analysis performed at the previous step, a global system of equation is created as

$$\underbrace{\begin{bmatrix} F \end{bmatrix}_{glob}}_{\begin{bmatrix} F \end{bmatrix}_{1}^{meas}} = \underbrace{\begin{bmatrix} D \end{bmatrix}_{glob}}_{\begin{bmatrix} D(\rho_{1},\lambda_{1},\alpha_{1}) \end{bmatrix}_{1}} \\ \vdots \\ \begin{bmatrix} F \end{bmatrix}_{n}^{meas} \end{bmatrix} = \underbrace{\begin{bmatrix} D(\rho_{n},\lambda_{n},\alpha_{n}) \end{bmatrix}_{n}}_{\begin{bmatrix} D(\rho_{n},\lambda_{n},\alpha_{n}) \end{bmatrix}_{n}} \cdot \{K_{glob}\}$$
(12)

 $[F]_i^{meas}$ is the vector containing the cutting forces for the i^{th} cutting test in the database, $[D(\rho_i, \lambda_i, \alpha_i)]_i$ is the matrix D computed in the previous step. This overdetermined system is solved using the Moore-Penrose pseudo-inverse to get a global set of parameters K_{glob} for the cutting force model.

2.5. Optimisation loop

In this final step, the initial guesses for ρ_i , λ_i and α_i are refined using an optimisation procedure. The objective is to minimise the RMS value of the difference between simulated and measured signal for the whole database. Nelder Mead algorithm [11] is used for this final step.

3. Application of the method

3.1. Experimental plan

A database of cutting tests on a workpiece in titanium (Ti6Al4V) has been used for the validation of the approach. The tool is a 2 mm diameter cylindrical endmill with 2 teeth with an helix angle of 20° (reference SECO 512020Z2.0-SIRON-A). A static dynamometer (Kistler MiniDyn 9256C2) recorded the cutting forces. The experimental set up is visible on figure 5.



Fig. 5. Experimental setup

Cutting tests have been carried out using two different cutting configurations: Half immersion downmilling $(a_e = D/2)$ and slotting $(a_e \ge D)$. No lubrication has been used during the machining. These cutting tests have been carried on a range of cutting speeds and feed per tooth around the values provided by the supplier of the tool (ensuring compatibility of cutting conditions with the tool/material couple and the edge radius of the cutter.):

- for half immersion downmilling
 - Cutting speed of 60, 75 and 90 m/min
 - Feed per tooth of 0.008, 0.009, 0.01, 0.011 and 0.012 mm/tooth
- for slot milling:

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- Cutting speed of 48, 60 and 72 m/min
- Feed per tooth of 0.0048, 0.0054, 0.006, 0.0066 and 0.0072 mm/tooth

The axial depth of cut was selected to ensure stable cutting conditions, minimizing cutter vibrations that could interfere with the identification process. All combinations have been tested, most of them repeated twice to get 57 individual tests. Cutting force signal has been filtered with a Butterworth low-pass filter (4th order, cutoff frequency of 2000 Hz) to take into account the bandwidth of the sensor.

Four cases have been compared in this study:

- the identification of the cutting coefficients for each test individually which give a lower bound of the error between model and measurement but no predictive value to the model;
- the use of a single set of cutting condition in the middle of the range of all cutting parameters tested is used for the identification ;
- the global optimisation proposed in this paper without the final step of optimisation;
- the global optimisation proposed in this paper with the final optimisation step.

The analysis of the results will focus on the components of the cutting force acting in the plane perpendicular to the spindle axis (x along feed and y perpendicular to this direction) considering that these two components are the most crucial to characterise cutting dynamics in classical machine tools. Force along the axis of the cutter will be presented on the graphs but not commented. Table 1 shows the analysis of the RMS difference between the measured and computed forces in x and y direction for the whole database.

	mean	min	max
	RMS (N)	RMS (N)	RMS (N)
Individual fit	7.75	4.20	14.31
Fit on a single test	10.53	4.22	15.81
Global coefficient	9.99	4.75	14.82
Global optimized	9.83	4.70	14.79

Table 1. RMS values for the four cases studied in the paper

3.2. Individual fit

While performing the identification on each cutting force signal individually, the specific pressures exhibit a large variation with K_r ranging from 2000 to 3150 MPa and K_t ranging from 1500 to 3450 MPa (figure 6) showing that this approach is not suitable to get a predictive model. The high ratio K_r/K_t may be attributed to the significant friction between the tool and the titanium alloy, exacerbated by the dry cutting conditions. The RMS value of the difference between measured and simulated signal ranges from 4.20 to 14.31 N. Figure 6 show the dispersion of the identified coefficients.

3.3. Fit on a single test

A classical approach used in the literature is to use a set of cutting parameters in the middle of the range for the identification of the cutting coefficients. For this paper, half immersion downmilling with a cutting speed of 75 m/min and a feed per tooth of 0.008 mm/tooth is selected. The fit is obviously the best for this set of cutting condition (see figure 7). The impact of cutter runout is clearly visible on the graph and the identification procedure is able to catch it efficiently.

This approach leads to higher mean deviation as compared to the previous case (10.53 N vs 7.75 N). In addition, a larger maximum error can be experienced for some of the cutting condi-



Fig. 6. Cutting force model parameters identified for each cutting tests individually.



Fig. 7. Cutting forces along feed (blue), lateral (red) and axial (green) direction. Plain lines are the measured values, lines with markers are the simulated values

tions. For example figure 8 show the comparison between measured and simulated signal for a cutting test in half immersion downmilling with a cutting speed of 90 m/min and a feed of 0.012 mm/min which is far from the reference condition. An important discrepancy (for example for the maximum value of the force) can be seen on the graph.

3.4. Global identification

The time needed for the optimisation is fairly short (15 minutes on a laptop with a I3-2310M CPU (2.10 GHz) and 4 Gb of RAM). This approach reduces the mean and maximum deviation as compared to the model using the cutting coefficients identified on a single cutting test (9.99 N vs 10.53 N for the mean value and 14.82 N vs 15.81 N for the maximum value) and provides a strating point for the final optimisation loop.



Fig. 8. Cutting forces along feed (blue), lateral (red) and axial (green) direction. Plain lines are the measured values, lines with markers are the simulated values

3.5. Global optimisation

The final optimisation unlocks the parameters with a nonlinear effect on the model. This last phase is the most computationally intensive of the procedure, adding around 45 minutes of computation before reaching convergence. This leads to the final identification of parameters of the cutting force law (K_t =2577 MPa, K_r =2139 MPa). This last step of optimisation allows a marginal gain in precision for the model (around 1 % improvement for all indicators) but strengthens the confidence in the model. For example, figure 9 shows the comparison between measured and simulated forces for the slotting case at a cutting speed of 60 m/min and 0.006 mm/tooth feed.



Fig. 9. Cutting forces along feed (blue), lateral (red) and axial (green) direction. Plain lines are the measured values, lines with markers are the simulated values

4. Conclusions

This paper presents a method used to identify the cutting parameters model coefficients as well as variables needed to model cutting forces in milling that are difficult to measure in practical conditions (initial shift of the cutter, runout,...). A global optimisation loop allows the identification of coefficients valid for a set of cutting tests using different cutting parameters to ensure that the identified model remains valid along a wider range of cutting conditions. On a set of milling tests performed on Ti6Al4V, this method allows identifying a model whose precision is close to the results obtained by setting a dedicated model for each cutting test.

The proposed approach is generic and could be used to identify cutting coefficients while machining with more complex tool geometry. The identification of more complex cutting force models is also possible within the same framework of identification.

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